Note on Algebras of Bounded Representation Type II

Dedicated to Prof. K. Asano on his sixtieth birthday.

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Let A be an associative algebra with a unit element over a field K and let $A = \sum_{\kappa} \sum_{i} Ae_{\kappa i}$ be a direct decomposition of A into directly indecomposable left ideals where $Ae_{\kappa i} \cong Ae_{\kappa 1} = Ae_{\kappa}$. Especially in this paper we suppose that the square of the radical of A is 0. In my paper [I] we studied necessary and sufficient conditions for an algebra to be of Bounded Representation Type when the square of the radical is 0 and K is algebraically closed[1]. In this paper these conditions are called B-conditions. In the same paper [I] it was shown that if K is not algebraically closed then B-conditions can not always be the necessary conditions. But in this paper we shall show that B-condions are sufficient conditions for A to be of Bounded Representation Type even if K is not algebraically closed. Namely we shall prove the following theorem.

Theorem An algebra A is of Bounded Representation Type if it satisfies the following conditions.

Suppose that $\{Ne_1, Ne_2, \cdots\cdots Ne_r\}$ $(\{e_1N, e_2N, \cdots\cdots, e_rN\})$ is a chain[2] such that at most one of Ne_i is the direct sum of three simple components not isomorphic to each other and other Ne_j are the direct sum of at most two simple components not isomorphic to each other.

(1) If Ne_i (e_iN) $(i=1,\cdots\cdots,r)$ are the direct sum of at most two simple components then they are the direct sum of two simple components except Ne_i (e_iN) $(i=1, r)$.

(2) If Ne_1 (e_1N) is the direct sum of three simple components then Ne_i (e_iN) $(i \neq 1, r)$ are the direct sum of two simple components and Ne_r (e_rN) is simple.

(3) If Ne_λ $(e_\lambda N)$ $(\lambda \neq 1, r)$ is the direct sum of three simple components then $r=3$ and Ne_i (e_iN) $(i=1, 3)$ are simple.

Proof We may prove this theorem in the following cases.

[**The case 1**]. Suppose that $Ne_1 = Au_{11} \oplus Au_{12} \oplus Au_{13}$ and $Ne_i = Au_{i1} \oplus Au_{i2}$ $(i=2,\cdots\cdots,r)$ where each Au_{ij} is not isomorphic to each other except $Au_{13} \cong Au_{21}$ and $Au_{i2} \cong Au_{(i+1)1}$.

[**The case 2**]. $Ne_1 = Au_{11}$, $Ne_2 = Au_{21} \oplus Au_{22} \oplus Au_{23}$ and $Ne_3 = Au_{31}$ where each

Au_{ij} is not isomorphic to each other except $Au_{11}\cong Au_{21}$ and $Au_{23}\cong Au_{31}$.

[**The case 3**]. $Ne_1=Au_{11}$. $Ne_2=Au_{21}$ and $Ne_i=Au_{i1}\oplus Au_{i2}$ $(i=3,\cdots\cdots,r)$ where each Au_i is not isomorphic to each other except $Au_{11}\cong Au_{21}\cong Au_{31}$ and $Au_{i2}\cong Au_{(i+1)1}$.

[**The case 4**]. $Ne_1=Au_{11}\oplus Au_{12}$, $Ne_2=Au_{21}$ and $Ne_3=Au_{31}\oplus Au_{32}$ where each Au_{ij} is not isomorphic to each other except $Au_{12}\cong Au_{21}\cong Au_{31}$.

[**The case 1**] From the results of my paper [II] an arbitrary A-left module $\mathfrak{M}$ is assumed to be

$$\{\overset{\oplus}{\Sigma} Ae_1m_{1ia}\oplus\overset{\oplus}{\Sigma} Ae_1m_{1ic}\oplus\overset{\oplus}{\Sigma} Ae_1m_{1id}\oplus\overset{\oplus}{\Sigma} Ae_1m_{1if}\oplus\overset{\oplus}{\Sigma}(Ae_1m_{1ig}+Ae_1m_{1ih})\}$$
$$+\sum_{\lambda}\{\overset{\oplus}{\Sigma} Ae_\lambda m_{\lambda ia}\oplus\overset{\oplus}{\Sigma} Ae_\lambda m_{\lambda ib}\}$$

where $Ae_1m_{1ia}\cong Ae_1$, $Ae_1m_{1ic}\cong Ae_1/Au_{11}$, $Ae_1m_{1id}\cong Ae_1/Au_{12}$, $Ae_1m_{1if}\cong Ae_1/Au_{11}+Au_{12}$, $Ae_1m_{1ig}=Ae_1/Au_{13}$, $Ae_1m_{1ih}\cong Ae_1m_{1ic}$, $Au_{12}m_{1ig}=Au_{12}m_{1ih}$, $Ae_\lambda m_{\lambda ia}\cong Ae_\lambda$ and $Ae_\lambda m_{\lambda ib}\cong Ae_\lambda/Au_{\lambda 2}$. In this case we can omit Ae_1m_{1ib}, Ae_1m_{1ie} and Ae_1m_{1ik} such that $Ae_1m_{1ib}\cong Ae_1/Au_{13}$, $Ae_1m_{1ie}=Ae_1/Au_{12}+Au_{13}$ and $Ae_1m_{1ik}\cong Ae_1/Au_{11}+Au_{13}$ because they are direct components of $\mathfrak{M}$.

First of all we denote $m_{2i\xi}$ such that $u_{13}m_{1j\eta}=\alpha(1j\eta,\ 2i\xi)u_{21}m_{2i\xi}(\eta=a,c,d,f,h;\xi=a,b$ by $m_{2i\xi\eta}$ where if $Au_{21}\cong\bar{A}\bar{e}_\rho$ then $\alpha(1jh,\ 2i\xi)\in\bar{e}_\rho A\bar{e}_\rho$.[3] Generally we denote $m_{\lambda i\xi}$ such that $u_{13}m_{1j\eta}=u_{21}m_{2ja\eta}$, $u_{22}m_{2ja\eta}=u_{31}m_{3ja\eta},\cdots\cdots$, $u_{(\lambda-1)2}m_{(\lambda-1)ja\eta}=u_{\lambda 1}m_{\lambda j\xi}$ by $m_{\lambda j\xi\eta}$ and such $m_{1j\eta}$ is denoted by $m_{1j\eta}'$.

Now suppose that $u_{21}m_{2ia}=\sum_{j,\eta}\alpha(2ia,\ 1j\eta)u_{13}m_{1j\eta}+\sum_{j,\eta}\alpha(2ia,\ 2j\eta)u_{21}m_{2j\eta}$.

If $\alpha(2ia,\ 1ja)\neq 0$ then $u_{21}m_{2iaa}=u_{13}m_{1ia}'$ by putting $m_{1ia}'=\sum_{j,\eta}\beta(2ia,1j\eta)\,m_{1j\eta}$ and $m_{2iaa}=m_{2ia}-\sum_{j,\eta}\beta(2ia,\ 2j\eta)\,m_{2j\eta}$ where $\alpha(2ia,\ 1j\eta)\,u_{13}=u_{13}\ \beta(2ia,1j\eta)$ and from now on the same notation is used.

If $\alpha(2ia,\ 1ja)=0$, $\alpha(2ia,\ 1ja')=0$, $\alpha(2ia,\ 1jd)\neq 0$ and $\alpha(2ia,\ 1jc)\neq 0$ then by the same way as above we can put $u_{21}m_{2iacd}=u_{13}m_{1ic}''+u_{13}m_{1id}''$ where $m_{2iacd}=m_{2ia}-\Sigma$ $\beta(2ia,\ 2j\eta)m_{2j\eta}$, $m_{1ic}''=\sum_{\eta\neq d}\beta(2ia,\ 1i\eta)\ m_{1i\eta}$ and $m_{1id}''=\sum_{\eta=d}\beta(2ia,\ 1i\eta)m_{1i\eta}$.

If $\alpha(2ia,\ 1ja)=0$ and $\alpha(2ia,\ 1ja')\neq 0$ then we can reduce this case to the above case because $u_{13}m_{1ja}'=u_{21}m_{2j\xi}(\xi=a,b)$.

Next suppose that $u_{21}m_{2ib}=\sum_{i,\eta}\alpha(2ib,\ 1i\eta)\,u_{13}m_{1i\eta}+\sum_{i,\xi}\alpha(2ib,\ 2i\xi)\,u_{21}m_{2i\xi}+\sum_{j}\alpha(2ib,$ $2i\xi\eta)\ u_{21}m_{2i\xi\eta}$.

If $\alpha(2ib,\ 2ia)\neq 0$ then we can replace m_{2ib} by m_{2ia} and this case is reduced to the above case.

If $\alpha(2ib,\ 2ia)=0$ and $\alpha(2ib,\ 2iaa)\neq 0$ then $u_{21}m_{2iaa}=u_{13}m_{1ia}'$ by putting $m_{1ia}'=\sum_{i,\eta}\beta(2ib,\ 2i\eta a)m_{1ia}'+\sum_{i,\xi,\eta}\beta(2ib,\ 2i\eta\xi)m_{1i\xi}'+\sum_{i,\xi}\beta(2ib,\ 1i\xi)m_{1i\xi}$ and $m_{2iaa}=\sum_{j}\beta(2ib,\ 2jb)$ $m_{2jb}+\Sigma\ \beta(2ib,\ 2i\eta a)m_{2i\eta a}+\Sigma\ \beta(2ib,\ 2i\eta\xi)m_{2i\eta\xi}$.

If $\alpha(2ib,\ 2ia)=0$, $\alpha(2ib,\ 2iaa)=0$ and $\alpha(2ib,\ 2iad)\neq 0$ or $\alpha(2ib,\ 2iac)\neq 0$ then by the same way as above we have $u_{21}m_{2iad}=u_{13}m_{1id}'$ and $u_{21}m_{2iac}=u_{13}m_{1ic}'$ where $u_{22}m_{2iad}$ $=u_{22}m_{2iac}$.

Thus continuing this operations $\mathfrak{M}$ is the direct sum of directly indecomposable A-left modules of following types:

(1) $Ae_1m_{1i\xi}$ $(\xi=a,b,c,d,e,f,k)$.

(2) $Ae_1m_{1ig}+Ae_1m_{1ih}$.

(3) $Ae_1m_{1i\xi}+Ae_2m_{2ia}+\cdots\cdots+Ae_\mu m_{\mu ia}$

where $u_{13}m_{1i\xi}=u_{21}m_{2ia}$, $u_{j2}m_{jia}=u_{(j+1)1}\ m_{(j+1)ia}(j=2,\cdots\cdots,\mu-1\ ;\mu=2,\cdots\cdots,r\ ;$ $\xi=a,d,c,f,h.)$

This module is denoted by $M_{\mu\xi}$.

(4) $M_{\mu a}+M_{\mu c}+Ae_{\mu+1}\ m_{(\mu+1)ia}+\cdots\cdots+Ae_\nu m_{\nu ia}$

where $u_{(\mu+1)1}m_{(\mu+1)ia}=u_{\mu2}m_{\mu ia}+u_{\mu2}m_{\mu ic}$ and $u_{\eta2}m_{\eta ia}=u_{(\eta+1)1}m_{(\eta+1)ia}$ $(\eta=\mu+1,$ $\cdots\cdots,\ \nu-1)$.

(5) $M_{\mu a}+M_{\mu c}+Ae_{\mu+1}\ m_{(\mu+1)ia}+Ae_{\mu+1}m_{(\mu+1)ia}+Ae_{\mu+2}m_{(\mu+2)ia}+Ae_{\mu+3}m_{(\mu+3)ia}+\cdots\cdots$ $+Ae_\nu m_{\nu ia}$

where $u_{(\mu+1)1}m_{(\mu+1)ia}=u_{\mu2}m_{\mu id}$, $u_{(\mu+1)1}m_{(\mu+1)(i+1)a}=u_{\mu2}m_{\mu ic}$, $u_{(\mu+1)2}m_{(\mu+1)ia}=u_{(\mu+1)2}$ $m_{(\mu+1)(i+1)a}=u_{(\mu+2)1}m_{(\mu+2)ia}$ and $u_{\eta2}m_{\eta ia}=u_{(\eta+1)1}m_{(\eta+1)ia}$ $(\eta=\mu+2,\cdots\cdots,\nu-1)$.

〔**The case 2**〕 In this case by the same way as the case 1 we may assume that an arbitrary A-left module $\mathfrak{M}$ is

$$\overset{\oplus}{\Sigma} Ae_1m_{1i}+\{\overset{\oplus}{\Sigma} Ae_2m_{2ia}\oplus\overset{\oplus}{\Sigma} Ae_2m_{2ib}\oplus\overset{\oplus}{\Sigma} Ae_2m_{2ic}\oplus\overset{\oplus}{\Sigma} Ae_2m_{2id}\oplus\overset{\oplus}{\Sigma} Ae_2m_{2ie}$$
$$\oplus\overset{\oplus}{\Sigma} Ae_2m_{2if}\oplus\overset{\oplus}{\Sigma}(Ae_2m_{2jg}+Ae_2m_{2jh})\}+\overset{\oplus}{\Sigma} Ae_3m_{3i}.$$

First of all we denoet m_{1i} such that $Au_{11}m_{1i}=Au_{21}m_{2i\xi}$ $(\xi=a,b,d,e,g)$ by $m_{1i\xi}'$ and $m_{2i\xi}$ by $m_{2i\xi}'$.

Now suppose that

$$u_{11}m_{1\lambda}=\Sigma\ \alpha(1\lambda,\ 2i\xi)u_{21}m_{2i\xi}+\Sigma\ \alpha(1\lambda,\ 2i\xi')u_{21}m_{2i\xi}'.$$

Then we may assume that $u_{11}m_{1\lambda}=\Sigma\ \alpha(1\lambda,\ 2i\xi)u_{21}m_{2i\xi}$ by replacing $m_{1\lambda}$ by $m_{1\lambda}-$ $\Sigma\beta(1\lambda,\ 2i\xi')m_{2i\xi}'$ because $u_{21}m_{2i\xi}'=u_{11}m_{1i\xi}'$.

If $\alpha(1\lambda,\ 2ia)\neq0$ then we have $u_{11}m_{1\lambda}=u_{21}m_{2ia}'$ by putting $m_{2ia}'=\sum_{i,\xi}\beta(1\lambda,\ 2i\xi)m_{2i\xi}$.

If $\alpha(1\lambda.\ 2ia)=0$, $\alpha(1\lambda,\ 2ib)\neq0$ and $\alpha(1\lambda,\ 2id)\neq0$ then we have $u_{11}m_{1\lambda}=u_{21}m_{2ib}''$ $+u_{21}m_{2id}''$ by putting $m_{2ib}''=\sum_{\xi\neq a,d}\beta(1\lambda,\ 2i\xi)m_{2i\xi}$ and $m_{2id}''=\sum_{\xi\neq a,d}\beta(1\lambda,\ 2i\xi)m_{2i\xi}$. In this way $\mathfrak{M}'=\Sigma Ae_1m_{1i}+\Sigma Ae_2m_{2i}$ is the direct sum of directly indecomposable A-left modules of following types:

(1) $M_1=Ae_1m_{1ia}'+Ae_2m_{2ia}'$.

(2) $M_2=Ae_2m_{2ia}$.

(3) $M_3=Ae_1m_{1id}'+Ae_2m_{2id}'$.

(4) $M_4=Ae_2m_{2id}$.

(5) $M_5=Ae_2m_{2ic}$.

(6) $M_6=Ae_1m_{1i}+(Ae_2m_{2ib}''\oplus Ae_2m_{2id}'')$where $u_{11}m_{1i}=u_{21}m_{2ib}''+u_{21}m_{2id}''$.

(7) $M_7=Ae_1m_{1i}+(Ae_2m_{2ig}'+Ae_2m_{2ih}')$where $u_{11}m_{1i}=u_{21}m_{2ig}'$ and $u_{22}m_{2ig}'=u_{22}m_{2ih}'$.

(8) $M_8=\ Ae_2m_{2ig}+Ae_2m_{2ih}$.

(9) $M_9=Ae_2m_{2i\xi}$ $(\xi=f,e,k,b)$.

Moreover continuing the same operation an arbitraryA-left module is the direct sum of directly indecomposable A-left modules of followimg types:

(1) M_i $(i=1,\cdots\cdots,9)$.

(2) $M_i+Ae_3m_{3i}$ $(i=1,\cdots\cdots,9)$. where $u_{23}m_{2i\xi}=u_{31}m_{3i}(\xi=a,d,c,f,h)$.

(3) $M_1+M_4+Ae_3m_{3i}$ where $u_{31}m_{3i}=\alpha(3i,\ 2ia')u_{23}m_{2ia}'+\alpha(3i,\ 2id)u_{23}m_{2id}$.

(4) $M_3+M_5+Ae_3m_{3i}$ where $u_{31}m_{3i}=\alpha(3i,\ 2id')u_{23}m_{2id}'+\alpha(3i,\ 2ic)u_{23}m_{2ic}$.

(5) $M_3+M_8+Ae_3m_{3i}$ where $u_{31}m_{3i}=\alpha(3i,\ 2id')u_{23}m_{2id}'+\alpha(3i,\ 2ih)\ u_{23}m_{2ih}$.

(6) $M_4+M_5+Ae_3m_{3i}$ where $u_{31}m_{3i}=\alpha(3i,\ 2id)u_{23}m_{2id}+\alpha(3i,\ 2ic)u_{23}m_{2ic}$.

(7) $M_5+M_6+Ae_3m_{3i}$ where $u_{31}m_{3i}=\alpha(3i,\ 2ic)u_{23}m_{2ic}+\alpha(3i,\ 2id'')u_{23}m_{2id}''$.

(8) $M_6+M_8+Ae_3m_{3i}$ where $u_{31}m_{3i}=\alpha(3i,\ 2id'')u_{23}m_{2id}''+\alpha(3i,\ 2ih)u_{23}m_{2ih}$.

〔**The case 3**〕 In this case an arbitrary A-left module is the direct sum of directly indecomposable A-left modules of following types:[4)]

(1) $Ae_3m_{3i}+\cdots\cdots+Ae_\mu m_{\mu i}$ $(\mu=3,\cdots\cdots,r)$ where $u_{\lambda 2}m_{\lambda i}=u_{(\lambda+1)1}m_{(\lambda+1)i}$.

(2) $Ae_1m'_{1i}+Ae_3m'_{3i}+\cdots\cdots+Ae_\mu m'_{\mu i}$ $(\mu=1,3,\cdots\cdots,r)$ where $u_{11}m'_{1i}=u_{31}m'_{3i}$ and $u_{\lambda 2}m'_{\lambda i}=u_{(\lambda+1)1}m'_{(\lambda+1)i}$. This module is denoted by $M_{1\mu}$.

(3) $Ae_2m''_{2i}+Ae_3m''_{3i}+\cdots\cdots+Ae_\mu m''_{\mu i}$ $(\mu=2,\ 3,\cdots\cdots,\ r)$ where $u_{21}m''_{1i}=u_{31}m''_{3i}$ and $u_{\lambda 2}m''_{\lambda i}=u_{(\lambda+1)1}m''_{(\lambda+1)i}$. This module is denoted by $M_{2\mu}$.

(4) $M_{1\mu}+M_{2\mu}+Ae_{\mu+1}\ m_{(\mu+1)i}+\cdots\cdots+Ae_\nu m_{\nu i}$ $(\mu=1,\ 2,\cdots\cdots,\ r\text{-}2;\ \nu=\mu+1,\cdots\cdots,r)$ where $u_{(\mu+1)1}m_{(\mu+1)i}=\alpha((\mu+1)i\,;\mu'i)u_{\mu 2}m'_{\mu i}+\alpha((\mu+1)i\,;\mu''i)u_{\mu 2}m''_{\mu i}$.

(5) $M_{1\mu}+M_{2\mu}+Ae_{\mu+1}m_{(\mu+1)i}+Ae_{\mu+1}\ m_{(\mu+1)(i+1)}+Ae_{\mu+2}m_{(\mu+2)i}+\cdots\cdots+Ae_\nu m_{\nu i}$ $(\mu=1,\ 2,\cdots\cdots,\ r\text{-}2;\ \nu=\mu+2,\ \cdots\cdots,r)$ where $u_{\mu 2}m'_{\mu i}=u_{(\mu+1)1}m_{(\mu+1)i}$; $u_{\mu 2}m''_{\mu i}=u_{(\mu+1)1}m_{(\mu+1)(i+1)}$ and $u_{(\mu+1)2}m_{(\mu+1)i}=u_{(\mu+1)2}m_{(\mu+1)(i+1)}$.

〔**The case 4**〕 In this case an arbitrary A-left module is the direct sum of directly indecomposable A-left modules of following types:

(1) $M_1=Ae_1m_{1ia}\cong Ae_1$.

(2) $M_2=Ae_1m_{1ib}\cong Ae_1/Au_{11}$.

(3) $M_3=Ae_2m_{2ia}\cong Ae_2$.

(4) $M_4=Ae_3m_{3ib}\cong Ae_3/Au_{32}$.

(5) $M_5=Ae_1m_{1ia}+Ae_3m_{3ia}$ where $u_{12}m_{1ia}=u_{31}m_{3ia}$.

(6) $M_6=Ae_1m_{1ia}+Ae_3m_{3ib}$ where $u_{12}m_{1ia}=u_{31}m_{3ib}$.

(7) $M_7=Ae_1m_{1ib}+Ae_3m_{3ia}$ where $u_{12}m_{1ib}=u_{31}m_{3ia}$.

(8) $M_8=Ae_1m_{1ib}+Ae_3m_{3ib}$ where $u_{12}m_{1ib}=u_{31}m_{3ib}$.

(9) $M_9=Ae_1m_{1ic}\cong Ae_1/Au_{12}$.

(10) $M_{10}=Ae_3m_{3ia}\cong Ae_3/Au_{31}$.

(11) $M_1+M_3+Ae_2m_{2i}$ where $u_{21}m_{2i}=\alpha(2i,\ 1ia)\ u_{12}m_{1ia}+\alpha(2i,\ 3ia)\ u_{31}m_{3ia}$.

(12) $M_1+M_4+Ae_2m_{2i}$ where $u_{21}m_{2i}=\alpha(2i,\ 1ia)\ u_{12}m_{1ia}+\alpha(2i,\ 3ib)\ u_{31}m_{3ib}$.

(13) $M_2+M_3+Ae_2m_{2i}$ where $u_{21}m_{2i}=\alpha(2i,\ 1ib)\ u_{12}m_{1ib}+\alpha(2i,\ 3ia)\ u_{31}m_{3ia}$.

(14) $M_2+M_4+Ae_2m_{2i}$ where $u_{21}m_{2i}=\alpha(2i,\ 1ib)\ u_{12}m_{1ib}+\alpha(2i,\ 3ib)\ u_{31}m_{3ib}$.

(15) $M_2+M_5+Ae_2m_{2i}$ where $u_{21}m_{2i}=\alpha(2i,\ 1ib)\ u_{12}m_{1ib}+\alpha(2i,\ 3ia)\ u_{31}m_{3ia}$.

(16) $M_2+M_6+Ae_2m_{2i}$ where $u_{21}m_{2i}=\alpha(2i,\ 1ib)\ u_{12}m_{1ib}+\alpha(2i,\ 1ia)\ u_{12}m_{1ia}$.

(17) $M_4+M_5+Ae_2m_{2i}$ where $u_{21}m_{2i}=\alpha(2i,\ 3ib)\ u_{31}m_{3ib}+\alpha(2i,\ 3ia)\ u_{31}m_{3ia}$.

(18) $M_4+M_7+Ae_2m_{2i}$ where $u_{21}m_{2i}=\alpha(2i,\ 3ib)\ u_{31}m_{3ib}+\alpha(2i,\ 3ia)\ u_{31}m_{3ia}$.

(19) $M_6+M_7+Ae_2m_{2i}$ where $u_{21}m_{2i}=\alpha(2i,\ 1ia)\ u_{12}m_{1ia}+\alpha(2i,\ 1ib)\ u_{12}m_{1ib}$.

Thus A is of bounded Representation Type.

Foot Note

(1) The latter half of the sixth condition of the theorem 2 in my paper [I] is corrected in the follewing way.

……, if $Ne_j(e_jN)$ $(j \neq 1,r)$ is the direct sum of three simple components then $\{Ne_1, Ne_2, Ne_3\}$ $(\{e_1N, e_2N, e_3N\})$ is a chain where Ne_i (e_iN) $(i=1,3)$ are simple and Ne_2 (e_2N) is the direct sum of three simple components.

The necessity of this condition can be proved by the same way as Lemma 6 if it is modified a little. Accordingly Lemma 7 is unnecessary.

(2) See my paper [I].

(3) $\alpha(1j\eta, 2i\xi)$ shows that it depends upon $m_{1j\eta}$ and $m_{2i\xi}$.

(4) This is proved similarly as the case 1 or 2.

References.

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II T. Yoshii: Note on Algebras of Bounded Representation Type I, Bull. of The Faculty of General Education, Gifu Univ., No. 4, 1968, 141–143.